Summary of the Content on the Math Subtest of the ACT (Pre-algebra and Statistics)

Fractions, Decimals and Percents:

- To change from a fraction to a decimal, divide bottom into top.
- To change from a decimal to a fraction, read as a fraction, write as a fraction, then reduce.
- To change from a decimal to a percent, (or vice versa) move the decimal 2 places. (Note: 1. = 100%)

Percent Problems:

Is means =, ***of*** means multiply.

What number, what is, and what percent refers to the variable.

$$\%\text{ increase or decrease} = \frac{\text{Change}}{\text{Original}}$$

Mult/Div Fractions:

Get rid of mixed numbers, cancel, then multiply straight across.

Add/Sub Fractions:

Get a common denominator first.

Order of Operation:

1. grouping symbols (work inside-out)
2. exponents
3. div. & mult. (work left to right)
4. subt. & add. (work left to right)

Evaluating Expressions:

Replace each letter by its value and simplify.

Simplify Expressions:

Distribute parentheses and then combine similar terms.

Scientific Notation:

Put a number in scientific notation by moving the decimal point until the number is between 1 and 10, and then multiply by the appropriate power.

Proportions:

Solve by cross-multiplying: $\frac{a}{b} = \frac{c}{d}$

$ad = bc$

Statistics:

- mean (average): Find by adding all the scores and dividing by number of scores.

(Note: If you know the average of a set of scores, you can find the total by multiplying the average by the number of scores.)

- median: Find by arranging the scores from high to low. The median is the score "in the middle." (Note: If there is an even number of scores, average the two scores in the middle.)
- mode: The most frequent score.

(Note: If more than one score is the most frequent, each of these scores is the mode.)

Probability:

The probability that something will occur =

$$\frac{\textit{The number of ways it can occur}}{\textit{The total number of possible occurrences}}$$

Example: 2 blue and 3 white balls are in a hat; the probability of drawing a blue ball =

$$\frac{\textit{number of blue balls}}{\textit{total number of balls}} = \frac{2}{5}$$

For any experiment, the sum of the probabilities of the outcomes must equal 1.

Fundamental Counting Principle:

With a series of choices, the total number of ways to choose is the product of the number of ways for each choice.

Summary of the Content on the Math Subtest of the ACT (Algebra)

Even & Odd Numbers: Any multiple of 2 is even.

even + even = even
odd + odd = even
even + odd = odd

even • even = even
even • odd = even
odd • odd = odd

Real Number Properties:

Natural Numbers: {1, 2, 3, ...}
Whole Numbers: {0, 1, 2, 3, ...}
Integers: {...-2, -1, 0, 1, 2...}
Rational Number: any number that ends or repeats when written as a decimal
Unit: {1}
Prime: {2, 3, 5, 7...}
Composite: {4, 6, 8, 9...}

commutative	$(a + b) = (b + a)$
associative	$(a + b) + c = a + (b + c)$
distributive	$a(b + c) = ab + ac$

Multiplying Polynomials:

$(x + a)(x + b) = x^2 + bx + ax + ab$
With more terms, multiply each member of the first group by each member of the second.

Rational Equations: (variables in the denominator):

Solve as you would any equation, except you must check:
Any value which makes the denominator = 0, makes the fraction undefined and, therefore, cannot be used as a solution.

Factoring:

1. Always take out the GCF (greatest common factor).

2. (2 terms) Look for the difference between the squares.
   $a^2 - b^2 = (a - b)(a + b)$

   Look for the difference or sum of cubes
   $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
   $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

3. (3 terms) General factoring (trial and error).

4. (4 terms) Factor by grouping.

Systems of Equations:

1. Solve by addition method: Multiply one or both equations by a number which, upon adding the equations, makes a variable drop out.

2. Solve using substitution: Solve for one variable in one equation. Substitute this expression for the variable in the other equation.

Exponents:

$$x^a \cdot x^b = x^{a+b}; \quad \frac{x^a}{x^b} = x^{a-b}; \quad (x^a)^b = x^{a \cdot b}$$

$$x^{-n} = \frac{1}{x^n}; \quad \frac{1}{x^{-n}} = x^n;$$

$$x^0 = 1; \quad -x^0 = -(1) = -1$$

Solving Equations:

1. Remove fractions
2. Distribute ( )
3. Combine similar terms
4. Get the variable on 1 side
5. Add & Subtract
6. Multiply & Divide

Square Roots:

Simplify: pull out as much as possible

Multiplication and Division:
Multiply Outsides and multiply insides.

Note: You can cancel outside pieces with outsides, insides with insides.

Addition and Subtraction:
Simplify first. Then add similar roots. (Insides must be the same.)

Solving Inequalities:

Solve using same procedure as equations. (Reverse inequality sign when multiplying or dividing by a negative.)

Open circle ***does not*** include endpoint, Closed circle does.

Approximate by Estimation:

$\sqrt{2} \approx 1.4$ $\quad$ $\sqrt{6} \approx 2.4$

$\sqrt{3} \approx 1.7$ $\quad$ $\sqrt{7} \approx 2.6$

$\sqrt{4} = 2$ $\quad$ $\sqrt{8} \approx 2.8$

$\sqrt{5} \approx 2.2$ $\quad$ $\sqrt{9} = 3$

Summary of the Content on the Math Subtest of the ACT (Geometry)

Triangle Theorems and Definitions:

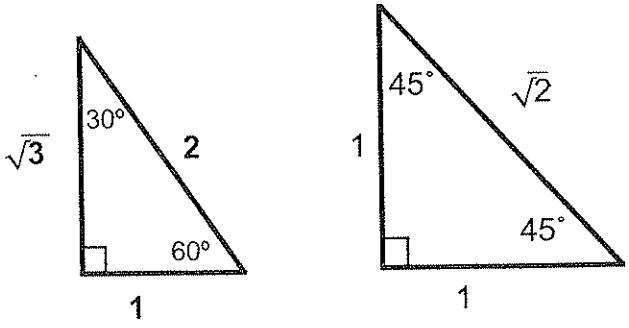
The ∠'s of a triangle add up to 180°.
Acute ∠ : less than 90°.
Right ∠ : 90°.
Obtuse ∠ : greater than 90°.
In any triangle, the biggest side is opposite the biggest ∠ , etc.

If 2 sides of a triangle are equal, then so are the angles opposite them, and vice versa. (Isosceles Triangle Theorem)

Any 2 sides of a triangle must, together, be longer than the third.

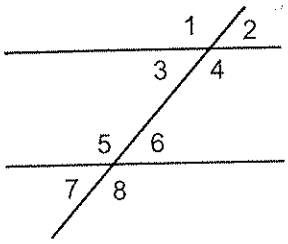
rt : $leg^2 + leg^2 = hyp^2$

3, 4, 5; 6, 8, 10 etc.
5, 12, 13; 10, 24, 26 etc.
Pythagorean triples



Special Triangles

Parallel Lines:



m ∠ 1 = m ∠ 5
m ∠ 2 = m ∠ 6 corresponding
m ∠ 3 = m ∠ 7 angles
m ∠ 4 = m ∠ 8

m∠3=m∠6 [alternate interior] m∠3 +m∠5 =180° [same side interior]
m∠5=m∠4 [alternate interior] m∠ 4 +m∠ 6 =180° [same side interior]

Vertical ∠ 's (∠ 1, ∠ 4) are always equal.
Two angles which make a line (∠ 5, ∠ 6) add up to 180°. (m∠ 5 +m∠ 6 = 180°)

Coordinate Geometry:

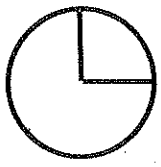
To find the slope / y-int. of a line, solve for y.
Y = mx + b m = slope b = y-int

$$Note: \ m = \frac{change\ in\ y}{change\ in\ x} = \frac{y_2 - y_1}{x_2 - x_1}$$

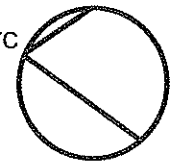
Parallel lines have equal slopes,
Perpendicular lines have negative reciprocal slopes.

Circles:

Central ∠ = arc



Inscribed ∠ = ½ arc



Circle = 360°

A radius is ⊥ to a tangent

Sum of the angles of a Polygon:

Sides	Sum
3	180°
4	360°
5	540°

(For each new side add 180°)
For regular polygons, all sides and angles are equal.

Formulas:

Area: Parallelogram: $A = b \cdot h$
(*Note: includes square and rectangle*)
Triangle: A = ½ *bh* Circle: A = r^2
Distance: (use Pythagorean Theorem)
Midpoint: (avg. of x's, avg. of y's)
Perimeter: Circle: C = 2 *r* = *D*
Volume: Box: V = *lwh* = *Bh*
(B = area of base)
Pyramid: Sphere:

$$V = \frac{1}{3}Bh \qquad V = \frac{4}{3}\pi r^3$$

Scale Factor: For any 2 similar figures (same shape) the ratio of any corresponding lengths is called the scale factor.
Note: Ratio lengths =SF, ratio areas = $(SF)^2$
In 3 dimensions: ratio volumes = $(SF)^3$

Similar Triangles:
Set up a proportion, solve by cross-multiplying.

Summary of the Content on the Math Subtest of the ACT (Advanced Alg. & Trig.)

Quadratic Equations: $ax^2 + bx + c = 0$

To solve: 1) Factor
2) Use quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

To find the vertex, minimum or maximum,
Step 1: let $x = \frac{-b}{2a}$
Step 2: plug this value in for x to find y.

Quadratic Relations:

Circle $(x-h)^2 + (y-k)^2 = r^2$
(h, k) = center r = radius

Ellipse $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$
(h,k) = center
Note: If the major axis is vertical, the larger number will be under the $(y-k)^2$

Parabola $y = a(x-h)^2 + k$
(h,k) = Vertex
$\pm a$ (opens up/down)
$x = a(y-k)^2 + h$
(h,k) = Vertex
$\pm a$ Opens right/left

Hyperbola $y = \frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

$y = \frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$

(h,k) = center

Laws of Logarithms:

Log A + Log B = Log AB

Log A – Log B = Log $\frac{A}{B}$

Absolute Value Equations:

$|x-a| < b$; $-b < x-a < b$

$|x-a| > b$; $x-a > b$ or $x-a < -b$

$|x-a| = b$; $x-a = b$ or $x-a = -b$

Roots and Radicals:

$$\sqrt[m]{x^n} = x^{\frac{n}{m}}$$

Discriminant:

$D = b^2 - 4ac$
$D < 0$: no real solutions (zeros)
$D = 0$: 1 real solution (zeros)
$D > 0$: 2 real solutions (zeros)

Trig Identities:

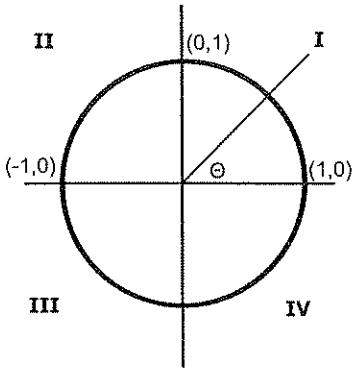
soh | cah | toa

$\sin\theta = \frac{Opp}{Hyp}$ $\cos\theta = \frac{Adj}{Hyp}$ $\tan\theta = \frac{Opp}{Adj}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\cot\theta = \frac{\cos\theta}{\sin\theta}$ $\sec\theta = \frac{1}{\cos\theta}$

$\csc\theta = \frac{1}{\sin\theta}$

$\sin^2\theta + \cos^2\theta = 1$ $\sin^2\theta = 2\sin\theta\cos\theta$



If (x,y) is the point where the terminal side hits the unit circle then:
$\sin\theta = y$
$\cos\theta = x$
$\tan\theta = y/x$

$f(x) = A\sin B(x-C) + D$
A = amplitude period = $\frac{2\pi}{B}$
C = horizontal shift D = vertical shift